# A Branch \& Cut algorithm for the Multi-trip Vehicle Routing Problem with Time Windows 

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#### Abstract

The Multi-trip Vehicle Routing Problem with Time Windows (MTVRPTW) generalizes the well-known Capacitated Vehicle Routing Problem (CVRP) in that vehicles can perform more than one trip within a maximum shift length but must comply customer time windows. The MTVRPTW has recently got the attention of scholars due to its applications to city logistics. A Branch \& Price approach is proposed in [1], while [2] tackles a variant with limited trip duration.

We propose a three-index MILP formulation for the MTVRPTW that makes use of base and replenishment arcs. The former model the direct connection between two nodes, while the latter imply a reload operation in between two clients. Base and replenishment arc variables are vehicle-indexed. Replenishment arcs allow to represent a journey as an elementary path and thus to ensure connectivity by separating SECs on a transformation of the graph. Further sets of two-indexed variables allow to impose time windows, shift length, and servicedependent loading time constraints.


The use of classical capacity constraints to enforce the load limit on vehicles leads to a Branch \& Cut algorithm. Capacity constraints are then strengthened after branching decisions to exploit some properties of the vehicle index.

Preliminary tests have been conducted, with promising results.
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